

NASA TECHNICAL NOTE



NASA TN D-3051

NASA TN D-3051

0130123



TECH LIBRARY KAFB, NM

LOAN COPY: RETURN
AFMIL (WLIL-2)
KIRTLAND AFB, NM

VOLUME AND SURFACE INSTABILITY IN SLIDING PLASMAS

by S. P. Talwar

*Goddard Space Flight Center
Greenbelt, Md.*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D. C.





0130123

VOLUME AND SURFACE INSTABILITY
IN SLIDING PLASMAS

By S. P. Talwar

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 - Price \$1.00

VOLUME AND SURFACE INSTABILITY IN SLIDING PLASMAS

by

S. P. Talwar

Goddard Space Flight Center

SUMMARY

The electrostatic instability arising due to a tangential discontinuity of velocity in two semi-infinite, homogeneous regions of a cold and collisionless plasma subject to a uniform magnetic field is investigated by use of two fluid equations. A general dispersion relation is derived allowing for interstreaming of charged particles in each region in addition to a slippage of one region over the other. Special cases are then discussed and the criteria for stability, monotonic instability and growing wave instability are derived.

CONTENTS

Summary	iii
INTRODUCTION.	1
EQUATIONS OF THE PROBLEM.	1
BOUNDARY CONDITIONS AND DISPERSION RELATION.	5
FIELD-FREE SLIDING PLASMAS	6
SLIDING PLASMAS WITH A UNIFORM FIELD ALONG STREAMING	9
Electron Oscillations in a Single Plasma	10
Both Ions and Electrons Perturbed	11
SLIDING PLASMAS WITH TRANSVERSE MAGNETIC FIELD.	13
ACKNOWLEDGMENTS	15
References	15



VOLUME AND SURFACE INSTABILITY IN SLIDING PLASMAS

by

S. P. Talwar*

Goddard Space Flight Center

INTRODUCTION

Plasmas characterized by relative tangential motion between various layers are a common occurrence in many astrophysical situations, e.g., solar-wind magnetospheric boundary and coronal streamers moving through the solar wind. This had led some workers (References 1, 2, 3, and 4) to investigate, in recent years, the instability which arises due to a tangential discontinuity in velocities in hydromagnetic plasmas. These investigations made use of the collision-dominated single fluid hydromagnetic equations, although in a recent investigation (Reference 5) the corresponding problem for a plasma of weak collisions characterized by an anisotropic plasma pressure is worked out using the Chew, Goldberger, and Low equations (Reference 6).

Quite often (e.g., in enhanced emission of charged particles at the time of a solar flare) situations arise where a plasma, carrying a net volume current, penetrates another plasma thus leading to a configuration where one expects to find the joint effect of two-stream instability and Kelvin-Helmholtz instability. In the present paper we intend to investigate the instability of such a plasma configuration while neglecting the effects of finite temperature and collisions.

EQUATIONS OF THE PROBLEM

Consider two semi-infinite, homogeneous, electrically neutral plasmas slipping past one another at the common interface $z = 0$. Each plasma is regarded as cold and collisionless and subject to a uniform magnetic field parallel to the interface. The particles in each plasma region ($z < 0$ and $z > 0$) are assumed to move parallel to each other in the initial state and at constant speeds, U_{1j} and U_{2j} , respectively along the x -direction. Here j stands for either electrons or ions. The electrons and ions may move together in each plasma in which case there will be no initial current flowing in either plasma, and the configuration of sliding plasmas will correspond to the classical Kelvin-Helmholtz problem as applied to ionized gases. The electrons and ions may, however, be taken to move at different speeds (but each spatially constant) in each plasma region. In this case there will be present a contra-streaming of charged particles in the body of each plasma in addition to a tangential discontinuity of velocities for each charged species at the interface between the two

*On leave from the Department of Physics and Astrophysics, University of Delhi, India

plasmas. Here, the mean electric current is not zero in the initial state, and one should expect a simultaneous presence of both the contrastreaming instability (the volume instability) and the Kelvin instability (the surface instability). We shall neglect the self magnetic field arising due to the initial current as compared to the prevailing magnetic field of external origin. The prevailing magnetic field may, in general, be inclined at a certain non-zero angle (but still parallel to the interface) to the direction (x -axis) of free streaming of charged particles in the initial state. If the magnetic field is assumed along the direction of free streaming there will be no initial electric field in either medium. On the other hand, if the prevailing magnetic field is, in particular, taken normal to the free streaming, then each plasma region will be characterized by an initial constant electric field in a direction normal to both the streaming motion and the magnetic field, i.e., normal to the interface (the z -direction). In case the ions and the electrons are not moving together in such a configuration, we may reconcile this by arguing that electrons (streaming through stationary ions) acquire a steady drift velocity in the crossed \vec{E} and \vec{B} fields and that the ions do not partake in this motion. Such a situation may arise in various physical situations (e.g., in the ionosphere in the form of the "electrojet" where a strong Hall current flows perpendicular to the earth's magnetic field, and in PIG type discharges (Reference 7) in the laboratory).

To investigate the stability of the above mentioned configuration, we will develop the perturbation equations, for the initial state which is defined as follows:

$$\left. \begin{aligned} \vec{B}_0 &= [B_{0x}, B_{0y}, 0] \\ \vec{V}_{0j} &= [U_{0j}, 0, 0] \\ \rho_0 &= \text{constant} \\ p_0 &= 0 \end{aligned} \right\} \quad (1)$$

For a zero temperature plasma, in which collisions are disregarded, the two fluid perturbation equations are written as,

$$\left(\frac{\partial}{\partial t} + \vec{U}_{0j} \cdot \vec{\nabla} \right) \vec{u}_j = \frac{e_j}{m_j} \left(\vec{E} + \frac{\vec{u}_j \times \vec{B}_0}{c} + \frac{\vec{U}_{0j} \times \vec{b}}{c} \right), \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \vec{U}_{0j} \cdot \vec{\nabla} \right) n_j = -N_0 \vec{\nabla} \cdot \vec{u}_j, \quad (3)$$

$$\vec{\nabla} \cdot \vec{b} = 0, \quad (4)$$

$$\vec{\nabla} \times \vec{b} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}, \quad (5)$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \vec{b}, \quad (6)$$

and

$$\vec{\nabla} \cdot \vec{E} = 4\pi \sum e_j n_j. \quad (7)$$

Here \vec{u}_j and n_j denote, respectively the perturbation in velocity at a point and the number density of the j^{th} class of particles (electron and ion). The quantities \vec{b} , and \vec{j} denote the perturbation in the magnetic field and current density at a point, \vec{E} stands for the electric field at a point in the medium, and the equilibrium quantities are shown with a suffix '0'. For simplicity we shall, in what follows, restrict our discussion to electrostatic perturbations only so that we may regard the perturbation \vec{b} in magnetic field to be zero. Thus the last term in Equation 2 vanishes and Equation 6 is written as,

$$\vec{\nabla} \times \vec{E} = 0, \quad (8)$$

so that

$$\vec{E} = -\vec{\nabla}\psi. \quad (9)$$

By coupling Equations 7 and 9, we get

$$\nabla^2 \psi + 4\pi \sum e_j n_j = 0. \quad (10)$$

If we assume the components of perturbation to vary with x , y , z , and t as (some function of z) $\cdot \exp i(k_x x + k_y y - \omega t)$, we can rewrite Equations 2 and 3 as

$$\left(-i\omega + i\vec{k} \cdot \vec{U}_{0j}\right) n_j = -N_0 \vec{\nabla} \cdot \vec{u}_j \quad (11)$$

and

$$\left(-i\omega + i\vec{k} \cdot \vec{U}_{0j}\right) \vec{u}_j + \vec{\Omega}_j \times \vec{u}_j = -\frac{e_j}{m_j} \vec{\nabla}\psi, \quad (12)$$

where $\vec{\Omega}_j$ stands for the Larmor frequency $e_j \vec{B}/m_j c$ for the charged particles (electrons, $-e$ and ions, $+e$), and has components Ω_{jx} and Ω_{jy} which correspond, respectively, to the magnetic field components, B_{0x} and B_{0y} in x - and y -directions.

Using Equation 11 we rewrite the perturbation equation for the electric potential, ψ , as

$$\nabla^2 \psi - 4\pi N_0 \sum_j e_j \frac{\vec{\nabla} \cdot \vec{u}_j}{(-i\omega + i\vec{k} \cdot \vec{U}_{0j})} = 0. \quad (13)$$

We now have to evaluate an expression for $\vec{\nabla} \cdot \vec{u}_j$ in terms of ψ to be substituted in the above equation so as to obtain the final equation in a single perturbation quantity ψ .

Taking the divergence and the curl of Equation 12, and simplifying we obtain

$$\left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right] \vec{\nabla} \cdot \vec{u}_j = -\frac{e_j}{m_j} \left[(-i\omega + i\vec{k} \cdot \vec{U}_{0j}) \nabla^2 \psi + \frac{(\vec{\Omega}_j \cdot \vec{\nabla})(\vec{\Omega}_j \cdot \vec{\nabla} \psi)}{(-i\omega + i\vec{k} \cdot \vec{U}_{0j})} \right]. \quad (14)$$

Eliminating $\vec{\nabla} \cdot \vec{u}_j$ from Equations 13 and 14 we obtain

$$\left\{ 1 + \sum_j \frac{\omega_{p_j}^2}{\left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right]} \right\} \nabla^2 \psi = \sum_j \frac{\omega_{p_j}^2}{(\omega - \vec{k} \cdot \vec{U}_{0j})^2} \frac{(\vec{\Omega}_j \cdot \vec{\nabla})^2 \psi}{\left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right]}, \quad (15)$$

where ω_{p_j} denotes the plasma frequency $(4\pi N_0 e^2/m_j)^{1/2}$ for the j^{th} class of particles.

With the spatial dependence of perturbations envisaged, we may rewrite Equation 15 as,

$$\left\{ 1 + \sum_j \frac{\omega_{p_j}^2}{\left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right]} \right\} D^2 \psi = \left(k^2 \left\{ 1 + \sum_j \frac{\omega_{p_j}^2}{\left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right]} \right\} - \sum_j \frac{\omega_{p_j}^2 (\vec{k} \cdot \vec{\Omega}_j^2)}{(\omega - \vec{k} \cdot \vec{U}_{0j})^2 \left[\Omega_j^2 - (\omega - \vec{k} \cdot \vec{U}_{0j})^2 \right]} \right) \psi \quad (16)$$

where D stands for $\partial/\partial z$.

The above equation with $U_{0j} = 0$ and $D^2 = -k_z^2$ gives the results for electrostatic plasma oscillations in a homogeneous static plasma subject to a uniform magnetic field. In particular it leads to the following well-known results.

$$\omega^2 = \sum_p \omega_p^2 \quad (\text{Longitudinal propagation})$$

and

$$1 + \frac{\omega_{pe}^2}{\Omega_e^2 - \omega^2} + \frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2} = 0 \quad (\text{Transverse propagation}) \quad (17)$$

Again, we may deduce from Equation 16 the results of two-stream instability for a homogeneous plasma carrying a uniform magnetic field along the direction of streaming, (Reference 8) and transverse to the direction of streaming (Reference 9) by taking $\vec{U}_{0j} \neq 0$ and $D = 0$.

BOUNDARY CONDITIONS AND DISPERSION RELATION

For a configuration of two plasmas sliding past each other Equation 16 holds for both plasma regions ($z \gtrless 0$) with respective values for the physical parameters involved in the equation. The respective solutions of Equation 16, bounded at $z = \pm\infty$, for the two plasma regions $z < 0$ and $z > 0$ are written as,

$$\left. \begin{aligned} \psi_1(z) &= A e^{m_1 z} & (z < 0) \\ \psi_2(z) &= B e^{-m_2 z} & (z > 0) \end{aligned} \right\} \quad (18)$$

and

where m_1 and m_2 are chosen so as to have positive real parts, and are given by

$$m_i^2 = k^2 \left[1 - \frac{\sum \frac{\omega_{pji}^2 (\vec{k} \cdot \vec{\Omega}_{ji})^2 / k^2}{(\omega - \vec{k} \cdot \vec{U}_{ij})^2 [\Omega_{ji}^2 - (\omega - \vec{k} \cdot \vec{U}_{ij})^2]} }{1 + \sum \frac{\omega_{pji}^2}{\Omega_{ji}^2 - (\omega - \vec{k} \cdot \vec{U}_{ij})^2}} \right] \quad (19)$$

here $i = 1, 2$ for the two regions $z < 0$ and $z > 0$, respectively.

We need to satisfy appropriate boundary conditions at the common interface between the two plasmas. The conditions are:

(1) Equation 8 gives the boundary condition that ψ should be continuous across the interface. This yields

$$A = B. \quad (20)$$

(2) Equation 13, when integrated across the interface, yields the condition

$$D\psi_1 - \sum \frac{4\pi e_j N_{01}}{(-i\omega + i\vec{k} \cdot \vec{U}_{ij})} u_{jz_1} = D\psi_2 - \sum \frac{4\pi e_j N_{02}}{(-i\omega + i\vec{k} \cdot \vec{U}_{2j})} u_{jz_2}. \quad (21)$$

The expression for u_{jz} can be derived from Equation 12 written in component form. After some simplifications we obtain,

$$\left[(-i\omega + i\vec{k} \cdot \vec{U}_{0j})^2 + \Omega_{j1}^2 \right] u_{jz} = -\frac{e_j}{m_j} \left[(-i\omega + i\vec{k} \cdot \vec{U}_{0j}) D\psi + i(\vec{k} \times \vec{\Omega}_j)_z \psi \right]. \quad (22)$$

Substituting Equation 22 into Equation 21 and making use of Equations 18 and 20 we obtain

$$\begin{aligned} m_1 \left(1 + \sum \frac{\omega_{pj1}^2}{\left[\Omega_{j1}^2 - (\omega - \vec{k} \cdot \vec{U}_{1j})^2 \right]} \right) \\ - \sum \frac{\omega_{pj1}^2 (\vec{k} \times \vec{\Omega}_{j1})_z}{(\omega - \vec{k} \cdot \vec{U}_{1j}) \left[\Omega_{j1}^2 - (\omega - \vec{k} \cdot \vec{U}_{1j})^2 \right]} \\ + \sum \frac{\omega_{pj2}^2 (\vec{k} \times \vec{\Omega}_{j2})_z}{(\omega - \vec{k} \cdot \vec{U}_{2j}) \left[\Omega_{j2}^2 - (\omega - \vec{k} \cdot \vec{U}_{2j})^2 \right]} = -m_2 \left(1 + \sum \frac{\omega_{pj2}^2}{\left[\Omega_{j2}^2 - (\omega - \vec{k} \cdot \vec{U}_{2j})^2 \right]} \right). \quad (23) \end{aligned}$$

Here m_1 and m_2 are given by Equation 19 with necessary suffixes. Equation 23, with values for m_1 and m_2 substituted from Equation 19, constitutes the dispersion relation determining the stability of the configuration under investigation. It is rather unwieldy for discussion in the general case, and we shall therefore discuss some special cases taking the propagation vector \vec{k} parallel to the streaming motion. For a propagation vector transverse to the direction of streaming, the configuration is equivalent to a static state and is thus stable in the absence of any discontinuity in magnetic field and plasma density at the interface.

FIELD-FREE SLIDING PLASMAS

If both plasma regions ($z \leq 0$) are devoid of any magnetic fields, m_1 and m_2 are each equal to \vec{k} and the dispersion relation, Equation 23, reduces to

$$1 = \frac{1}{2} \frac{\omega_{pe1}^2}{\left(\omega - k U_{1e} \right)^2} + \frac{N_2/N_1}{\left(\omega - k U_{2e} \right)^2} + \frac{m}{M} \left(\frac{1}{\left(\omega - k U_{1i} \right)^2} + \frac{N_2/N_1}{\left(\omega - k U_{2i} \right)^2} \right), \quad (24)$$

where m and M , respectively, stand for electron and ion masses, and N_1 and N_2 are the particle densities in the two regions. In Equation 24 we have considered the propagation vector \vec{k} along the direction of streaming (namely the x -axis). For simplicity we may further restrict ourselves to a single homogeneous plasma ($N_1 = N_2$) wherein the ions are at rest and the electrons stream past them with equal and opposite speeds in the two semi-infinite regions $z < 0$ and $z > 0$. Thus, with

$U_{1i} = U_{2i} = 0$ and $U_{1e} = -U_{2e} = U_0$, Equation 24 reduces to

$$1 = \frac{1}{2} \omega_{pe}^2 \left[\frac{1}{(\omega - k U_0)^2} + \frac{1}{(\omega + k U_0)^2} + 2 \frac{m}{M \omega^2} \right]. \quad (25)$$

This is a cubic equation in ω^2 which does not possess any negative real root for ω^2 leading to monotonic instability in case the inequality $k^2 U_0^2 > \omega_{pe}^2$ is satisfied. Equation 24 may be compared with the corresponding formula for the two-stream instability in a homogeneous current-carrying plasma in the absence of any magnetic field. Measuring ω , $k U_0$ in units of ω_{pe} we may rewrite the Equation 25 as

$$F_0(x, y) = 1 - \left[\frac{\epsilon}{x^2} + \frac{1}{(x - y)^2} \right] + \frac{2xy}{(x^2 - y^2)^2} = 0, \quad (26)$$

where

$$x = \frac{\omega}{\omega_{pe}}, \quad y = \frac{k U_0}{\omega_{pe}} \text{ and } \epsilon = \frac{m}{M}. \quad (27)$$

It may be noted that our Equation 26 differs in the last term on the left hand side from the conventional formula for two-stream instability in a homogeneous plasma devoid of any relative slippage between its two halves. The function, $F_0(x, y)$, and the corresponding function, $F(x, y)$ without the last term $2xy/(x^2 - y^2)^2$, are plotted in Figures 1 and 2 for $y = 0.5$ and 1.5 respectively. The curve $F_0(x, y)$, symmetric in x , is always less negative than the curve $F(x, y)$. For $y > 1$, the plot of both functions F and F_0 demonstrates that all roots (four for F and six for F_0) are real, leading to stable oscillatory modes. For $y < 1$, on the other hand, the function F corresponds to an overstable situation (instability through growing waves) whereas the function F_0 leads to a monotonically unstable situation corresponding to ω^2 a negative real quantity. Let us now assume that the ions and the electrons move together so that there is no initial current in either plasma region. In this case, the dispersion relation, Equation 24, reduces to,

$$1 = \frac{1}{2} \left[\frac{\omega_{p1}^2}{(\omega - k U_1)^2} + \frac{\omega_{p2}^2}{(\omega - k U_2)^2} \right], \quad (28)$$

where

$$\omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2 \simeq \omega_{pe}^2.$$

There is no contraststreaming of particles in the body of either plasma, and the only discontinuity in motion is present at the interface. Equation 28 for $\omega_{p1} = \omega_{p2}$ and $U_1 = -U_2 = U_0$

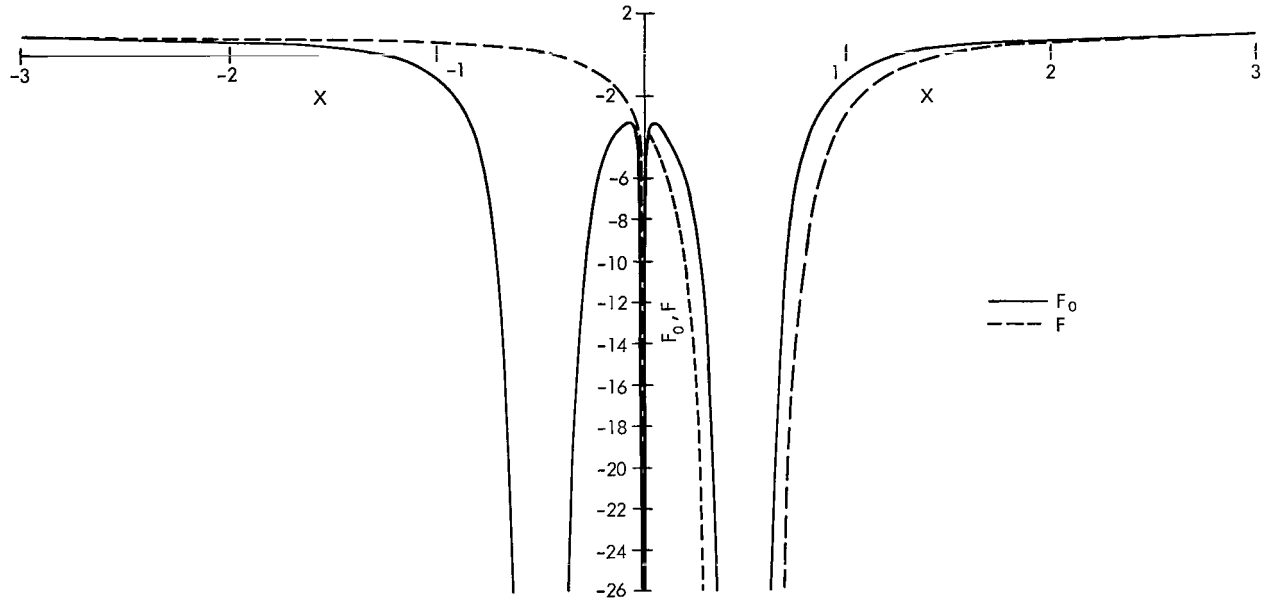


Figure 1—Plot of the functions $F_0(x, y)$ and $F(x, y)$ versus x for $y = 0.5$.

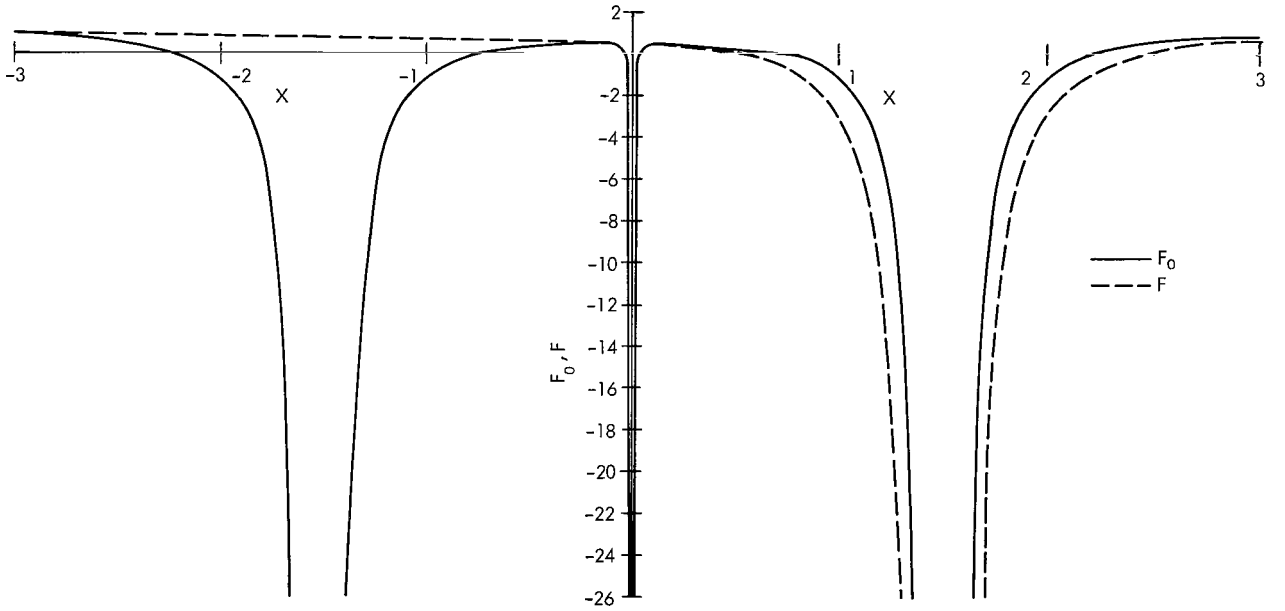


Figure 2—Plot of the functions $F_0(x, y)$ and $F(x, y)$ versus x for $y = 1.50$.

yields,

$$\frac{\omega^2}{\omega_{pe}^2} = \frac{1}{2} \left[1 \pm \left(1 + \frac{8k^2 U_0^2}{\omega_{pe}^2} \right)^{1/2} \right] + \frac{k^2 U_0^2}{\omega_{pe}^2}. \quad (29)$$

This formula is identical to the one obtained by Barston (Reference 10) and shows that a tangential discontinuity in a field-free homogeneous plasma is stable or unstable monotonically depending on

whether the relative speed $2U_0$ is more or less than the critical value defined by

$$(kU_0)_* = \omega_{pe} \quad (30)$$

It may be noted that the classical Kelvin-Helmholtz instability in hydrodynamics is recovered in the limit $\omega_{pe} \rightarrow \infty$. Further, the Equation 28 shows that there is no possibility of growing wave instability in the configuration. In Figure 3 we have shown in curve a, the dependence of the root ω^2/ω_{pe}^2 (corresponding to negative sign in Equation 28) on the parameter kU_0/ω_{pe} . For a given tangential speed the wave numbers of perturbation lying between the range 0 and k_* (Equation 30) are monotonically unstable, there being a mode of maximum instability for an intermediate wave number defined by

$$k_{max} = \sqrt{\frac{3}{8}} \frac{\omega_{pe}}{U_0} \quad (31)$$

with maximum growth rate given by

$$\left| \frac{\omega^2}{\omega_{pe}^2} \right|_{max} = \frac{\omega_{pe}^2}{8} \quad (32)$$

SLIDING PLASMAS WITH A UNIFORM FIELD ALONG STREAMING

If we suppose that the prevailing magnetic field is parallel to both the direction of streaming of particles in both plasma regions $z \gtrless 0$ and the propagation vector \vec{k} , then the terms involving $(\vec{k} \times \vec{\Omega}_j)$ disappear in Equation 22. The dispersion relation, Equation 23, is thus written as,

$$\frac{m_1}{m_2} = - \frac{\left(1 + \sum \frac{\omega_{pj2}^2}{\Omega_{j2}^2 - \omega^2} \right)}{\left(1 + \sum \frac{\omega_{pj1}^2}{\Omega_{j1}^2 - \omega^2} \right)} \quad (33)$$

Substituting for m_1 and m_2 from Equation 18 we obtain the dispersion relation as,

$$\begin{aligned} & \left(1 - \sum \frac{\omega_{pj1}^2}{\Omega_{j1}^2} \right)^{1/2} \left(1 + \sum \frac{\omega_{pj1}^2}{\Omega_{j1}^2 - \omega_{j1}^2} \right)^{1/2} \\ &= - \left(1 - \sum \frac{\omega_{pj2}^2}{\Omega_{j2}^2} \right)^{1/2} \left(1 + \sum \frac{\omega_{pj2}^2}{\Omega_{j2}^2 - \omega_{j2}^2} \right)^{1/2} \end{aligned} \quad (34)$$

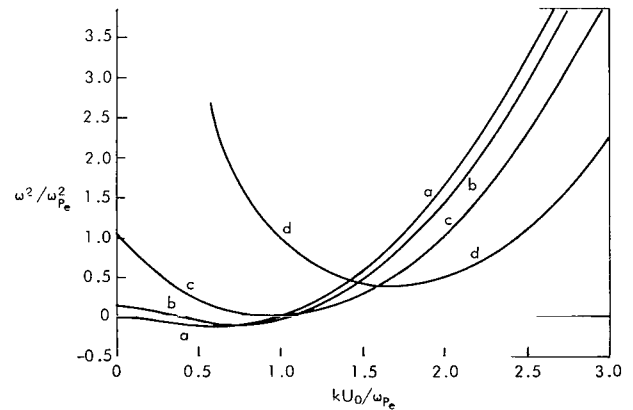


Figure 3—Plot of the dependence of the root ω^2/ω_{pe}^2 (from Equation 36) on the parameter kU_0/ω_{pe} . Curve a, $\Omega_e/\omega_{pe} = 0$; curve b, $\Omega_e/\omega_{pe} = 0.5$; curve c, $\Omega_e/\omega_{pe} = 1.0$; and curve d, $\Omega_e/\omega_{pe} = 2.0$.

where

$$\left. \begin{aligned} \omega_{1j} &= \omega - kU_{1j} \\ \omega_{2j} &= \omega - kU_{2j} \end{aligned} \right\} \quad (35)$$

and

Electron Oscillations in a Single Plasma

For convenience in handling the dispersion Equation 34 we may restrict our discussion to the case of electron oscillations alone and thus regard the ions to be unperturbed due to their heavy mass. In this case the summations in Equation 34 disappear and we get a simplified dispersion relation for a homogeneous plasma with a uniform magnetic field, so that the two halves of the plasma slide past each other along the field lines. This is written as,

$$\omega^4 - \left(2k^2 U_0^2 + \Omega_e^2 + \omega_{pe}^2\right) \omega^2 + \left[k^4 U_0^4 - \left(\Omega_e^2 + \omega_{pe}^2\right)\left(k^2 U_0^2 - \Omega_e^2/2\right)\right] = 0 \quad (36)$$

Here we have again taken $U_1 = -U_2 = U_0$ and cancelled a non-zero factor, $(\omega_{e2}^2 - \omega_{e1}^2)$, in the derivation of Equation 36.

Equation 36 suggests that the configuration envisaged here does not show any overstability (growing wave instability corresponding to ω^2 a complex quantity) in case the inequality $\Omega_e^2 < 2k^2 U_0^2$ is satisfied. The configuration is then stable or monotonically unstable depending on whether the inequality

$$\left. \begin{aligned} \left(\Omega_e^2 + \omega_{pe}^2\right) &< \frac{2k^4 U_0^4}{\left(2k^2 U_0^2 - \Omega_e^2\right)} \\ \text{or} \\ &> \frac{2k^4 U_0^4}{\left(2k^2 U_0^2 - \Omega_e^2\right)} \end{aligned} \right\} \quad (37)$$

is satisfied. The configuration will be overstable only if $\Omega_e > \omega_{pe}$ and the relative velocity satisfies the inequality

$$8k^2 U_0^2 < \left(\Omega_e^2 - \omega_{pe}^2\right) \quad (38)$$

The dependence of one root, ω^2 (in units of $\omega_{p_e}^2$), likely to show instability on kU_0 (measured in units of ω_{p_e}) is exhibited in Figure 3 for $\Omega_e/\omega_{p_e} = 0.5, 1.0$ and 2.0 (curves *b*, *c*, and *d* respectively). Curve *a* represents the case of zero magnetic field. Clearly the configuration with field is unstable monotonically for a range of relative speeds, given by the following expressions, in case the electron Larmor frequency is less than the plasma frequency.

$$\left. \begin{aligned} (kU_0)_1^* &= \left\{ \frac{(\Omega_e^2 + \omega_{p_e}^2)}{2} \left[1 - \left(\frac{\omega_{p_e}^2 - \Omega_e^2}{\omega_{p_e}^2 + \Omega_e^2} \right)^{1/2} \right] \right\}^{1/2} \\ \text{and} \\ (kU_0)_2^* &= \left\{ \frac{(\Omega_e^2 + \omega_{p_e}^2)}{2} \left[1 + \left(\frac{\omega_{p_e}^2 - \Omega_e^2}{\omega_{p_e}^2 + \Omega_e^2} \right)^{1/2} \right] \right\}^{1/2} \end{aligned} \right\} \quad (39)$$

The configuration is thus stable outside this range of relative speeds. The range of unstable relative speeds shrinks to zero as the electron Larmor frequency approaches the plasma frequency. Thereafter the configuration is either stable or exhibits overstability for low enough relative speeds. For $\Omega_e = 2\omega_{p_e}$, for example, the configuration is overstable up to $kU_0 \simeq 0.6 \omega_{p_e}$ and stable beyond.

Thus we conclude by saying that the effect of magnetic field along the streaming direction is stabilizing in that the range of unstable tangential speeds is narrowed and the growth rate for any unstable wavelength is suppressed, till beyond a critical value of magnetic field (corresponding to $\Omega_e = \omega_{p_e}$) the instability appears in the form of overstability for low tangential speeds only. For infinitely large magnetic fields, therefore, the configuration is unstable through overstability. The existence of instability for infinitely large magnetic fields was established earlier by Harrison and Stringer (Reference 11). Their analysis, however, did not apply to finite values of magnetic fields.

Both Ions and Electrons Perturbed

The assumption that only electrons are perturbed and not the ions is justified in case the doppler shifted frequencies, ω_1 and ω_2 , for either region are much larger than the ion cyclotron frequency. In general, ions also contribute and the general dispersion relation, Equation 33 is a 16th degree polynomial in ω , the parameter deciding the question of stability of the configuration. To reduce the degree of the dispersion relation when both ions and electrons are perturbed we may consider the following special situations.

No initial electric current in uniform plasma

If the ions and electrons are taken to move together along the magnetic field lines, the dispersion relation for the propagation vector along the streaming direction is written as,

$$\omega^4 - 2 \left[k^2 U_0^2 + \frac{\Omega_e^2 (\omega_{pe}^2 + \Omega_i^2)}{(\omega_{pe}^2 + \Omega_e^2)} \right] \omega^2 + \left[k^4 U_0^4 - \Omega_e^2 (2k^2 U_0^2 - \Omega_i^2) \frac{(\omega_{pi}^2 + \Omega_i^2)}{(\omega_{pe}^2 + \Omega_e^2)} \right] = 0 \quad (40)$$

Here we have taken the doppler-shifted frequencies $(\omega - kU_0)$ and $(\omega + kU_0)$ for the two plasma regions to be very much less than the electron cyclotron frequency, Ω_e . Thus, we find that in the approximation mentioned above, the configuration of slow relative slippage in a uniform current free plasma is stable in the presence of a uniform magnetic field, along the streaming motion, and for all tangential speeds except for a narrow range of tangential speeds lying between the values corresponding to the positive and negative signs before the square root term in the following expression.

$$(k^2 U_0^2)_{1,2} = \frac{m}{M} \frac{\Omega_e^2 (\omega_{pe}^2 + \frac{m}{M} \Omega_e^2)}{(\omega_{pe}^2 + \Omega_e^2)} \left\{ 1 \pm \left[1 - \frac{m}{M} \frac{(\omega_{pe}^2 + \Omega_e^2)}{(\omega_{pe}^2 + \frac{m}{M} \Omega_e^2)} \right]^{1/2} \right\} \quad (41)$$

It may be noted that there is no growing wave instability possible in this approximation.

Uniform plasma carrying an initial current

Let us now suppose that in the two halves of a single plasma permeated with a homogeneous magnetic field, the electrons stream past the stationary ions with equal and opposite speed along the direction of the prevailing magnetic field. In this case the dispersion formula, Equation 34, for the case of slow relative velocities reduces to,

$$\left\{ \omega^4 - 2\omega^2 (\Omega_e^2 + k^2 U_0^2) + \left[k^2 U_0^2 (k^2 U_0^2 - 2\Omega_e^2) + \Omega_e^4 \right] \right\} \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2 - \omega^2} \right) + \left\{ \omega^4 - 2\omega^2 (k^2 U_0^2 + \omega_{pe}^2) + \left[k^4 U_0^4 + \omega_{pe}^2 (\Omega_e^2 - 2k^2 U_0^2) \right] \right\} = 0 \quad (42)$$

This is a cubic in ω^2 and so must possess at least one real root, which for monotonic instability is required to be negative. A necessary condition for monotonic instability to arise is that the electron plasma frequency exceeds the electron Larmor frequency. In that case the configuration is unstable monotonically for a small range of relative speeds given by the expression

$$(k^2 U_0^2)_{1,2} = \frac{\left[\Omega_e^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2} \right) + \omega_{pe}^2 \right]}{\left(2 + \frac{\omega_{pi}^2}{\Omega_i^2} \right)} \left\{ 1 \mp \left[\frac{\Omega_i^2 (\omega_{pe}^2 - \Omega_e^2)}{\Omega_e^2 (\omega_{pi}^2 + \Omega_i^2) + \Omega_i^2 \omega_{pe}^2} \right]^{1/2} \right\} \quad (43)$$

It may be noted that in the case of electrons streaming through stationary ions, the monotonically unstable range of relative speeds (obtained from Equation 43) is very much less than the unstable range of speeds when ions are regarded as unperturbed (Equation 39).

The Equation 42 reduces to a quadratic equation in ω^2 for frequencies much less than the ion Larmor frequency and in this limit we conclude that the configuration is monotonically unstable for relative speeds in the range defined by Equation 43 and is unstable through overstability if the following inequality holds

$$k^2 U_0^2 < \frac{(\Omega_e^2 - \omega_{pe}^2)}{8 \left(1 + \frac{\omega_{pi}^2}{2\Omega_i^2}\right)}. \quad (44)$$

Clearly a necessary condition for growing wave instability to occur for low relative speeds is that the electron plasma frequency is less than the electron Larmor frequency. Thus, the configuration is monotonically unstable, though only for a narrow range of tangential speeds, if $\omega_{pe} > \Omega_e$; and overstable for low enough tangential speeds if $\omega_{pe} < \Omega_e$.

In Figure 4 we have shown the variation of one root (the less positive) of the quadratic equation in ω^2 , as obtained by neglecting ω^2 in comparison to Ω_i^2 in the term involving ω_{pi}^2 in Equation 42, with the parameter kU_0 for a few values of Ω_e . The curves *a*, *b*, *c* are respectively for $\Omega_e/\omega_{pe} = 0.5, 1$, and 2 . In the last value the configuration is overstable for relative speeds below a critical value defined by $kU_0 \simeq 0.6 \omega_{pe}$.

SLIDING PLASMAS WITH TRANSVERSE MAGNETIC FIELD

In case the two plasma regions ($z \lesssim 0$) are assumed to carry a uniform magnetic field transverse to the direction of streaming in each region and the propagation vector \vec{k} , and m_1 and m_2 (Equation 19) are each equal to k and the dispersion relation, Equation 23, is thus written as

$$2 - \sum \frac{\omega_{pi1}^2}{(\omega - kU_{1j}) [\Omega_{j1} + (\omega - kU_{1j})]} + \sum \frac{\omega_{pi2}^2}{(\omega - kU_{2j}) [\Omega_{j2} - (\omega - kU_{2j})]} = 0 \quad (45)$$

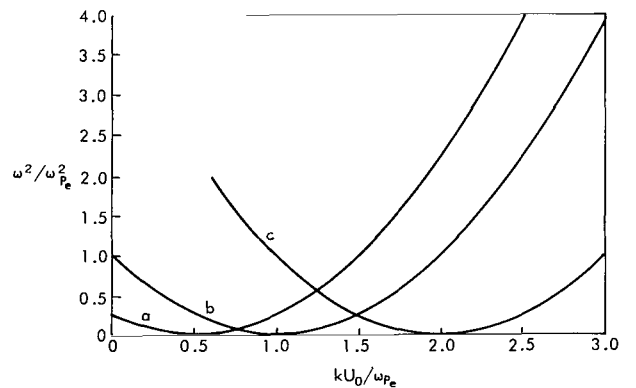


Figure 4—Plot of the dependence of the root ω^2/ω_{pe}^2 (from Equation 42) on the parameter kU_0/ω_{pe} . Curve *a*, $\Omega_e/\omega_{pe} = 0.5$; curve *b*, $\Omega_e/\omega_{pe} = 1.0$; and curve *c*, $\Omega_e/\omega_{pe} = 2.0$.

For electron oscillations alone in a single plasma it further reduces to a quadratic in ω^2 , given by

$$\omega^4 - \omega^2 \left[\omega_{pe}^2 + k^2 U_0^2 + (\Omega_e - kU_0)^2 \right] + kU_0 (\Omega_e - kU_0) \left[\omega_{pe}^2 + kU_0 (\Omega_e - kU_0) \right] = 0 . \quad (46)$$

Here we have taken $U_1 = -U_2 = U_0$.

It follows from the Equation 46 that the electron oscillations, in a uniform plasma with its two halves slipping past each other in a transverse uniform magnetic field, do not lead to growing wave instability. They do, however, show monotonic increase in amplitude in case the following conditions are simultaneously met with, i.e.,

$$\left. \begin{aligned} \Omega_e &< kU_0 \\ \omega_{pe}^2 &> kU_0 (kU_0 - \Omega_e) , \end{aligned} \right\} \quad (47)$$

and

so that the configuration is stable for all electron densities in case $kU_0 < \Omega_e$.

Alternatively, we could argue that the electron oscillations lead to a monotonic instability only for wave numbers of perturbation greater than a certain critical value given by the expression,

$$(kU_0)^* = \frac{\Omega_e}{2} \left[1 + \left(1 + \frac{4\omega_{pe}^2}{\Omega_e^2} \right)^{1/2} \right] . \quad (48)$$

If one incorporates mutual streaming of particles also, leading to an initial electric current in each region, the dispersion relation, Equation 45, complicated for the general case, reduces to a cubic in ω^2 for the case of electrons streaming through stationary ions in a single plasma with equal and opposite speeds in the two regions $z \gtrless 0$. This is written as,

$$\left[1 + \frac{\omega_{pi}^2}{(\Omega_i^2 - \omega^2)} \right] \left\{ \omega^4 - \omega^2 \left[2k^2 U_0^2 + (\Omega_e - kU_0)^2 \right] + k^2 U_0^2 (\Omega_e - kU_0)^2 \right\} - \omega_{pe}^2 (\omega^2 + k^2 U_0^2 - kU_0 \Omega_e) = 0 . \quad (49)$$

This equation gives the following condition for monotonic instability in the medium.

$$kU_0 > \frac{\Omega_e}{2} \left\{ 1 + \left[1 + \frac{4\omega_{pe}^2 \Omega_i^2}{\Omega_e^2 (\Omega_i^2 + \omega_{pi}^2)} \right]^{1/2} \right\} . \quad (50)$$

A comparison of the two equations, 48 and 50, shows that the critical wave number beyond which the configuration is unstable monotonically, is somewhat greater when perturbed motions of both electrons and ions are taken into account in a current carrying plasma than without it.

ACKNOWLEDGMENTS

The author is thankful to Dr. D. Tidman for helpful discussion.

This work was done while the author held a Senior Resident Research Associateship of the National Academy of Sciences - National Research Council.

(Manuscript received May 3, 1965)

REFERENCES

1. Fejer, J. A., "Hydromagnetic Stability at a Fluid Velocity Discontinuity between Compressible Fluids," *Phys. Fluids* 7(4):499-503, April 1964.
2. Parker, E. N., "Dynamical Properties of Stellar Coronas and Stellar Winds. III. The Dynamics of Coronal Streamers," *Astrophys. J.* 139(2):690-709, February 1964.
3. Sen, A. K., "Effect of Compressibility on Kelvin-Helmholtz Instability in a Plasma," *Phys. Fluids* 7(8):1293-1298, August 1964.
4. Talwar, S. P., "Hydromagnetic Stability of the Magnetospheric Boundary," *J. Geophys. Res.* 69(13):2707-2714, July 1, 1964.
5. Talwar, S. P., "Kelvin-Helmholtz Instability in an Anisotropic Plasma," Technical Document X-641-64-305, Washington: National Aeronautics and Space Administration, October 1964. (To appear in *Phys. Fluids*, June 1965)
6. Chew, G. F., Goldberger, M. L., and Low, F. E., "The Boltzmann Equation and the One-Fluid Hydromagnetic Equations in the Absence of Particle Collisions," *Proc. Roy. Soc. A* 236:112-118, July 10, 1956.
7. Lehnert, B., "Diffusion Processes in the Positive Column in a Longitudinal Magnetic Field," in: *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, held at Geneva, 1 September - 13 September, 1958. Controlled Fusion Devices*, New York: United Nations, 1958, Vol. 32, pp. 349-357.
8. Buneman, O., "Dissipation of Currents in Ionized Media," *Phys. Rev.* 115(3):503-517, August 1, 1959.
9. Buneman, O., "Instability of Electrons Drifting through Ions across Magnetic Field," Plasma Physics-Accelerators-Thermonuclear Research, Pt. C, *J. Nuclear Energy* 4(2):111-117, April 1962.

10. Barston, E. M., "Variational Technique for Determining Dispersion Relations in Plasma Systems," *Phys. Fluids* 6(6):828-833, June 1963.
11. Harrison, E. R., and Stringer, T. E., "Longitudinal Electrostatic Oscillations in Velocity Gradient Plasmas. II. The Adjacent-Stream Model," *Proc. Phys. Soc.* 82(Pt. 5):700-709, November 1963.

3/18/85
gj

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546